LAUTECH Journal of Civil and Environmental Studies
Volume 10, Issue 1; March, 2023

DOI: 10.36108/laujoces/3202.01.0130

Evaluation of Selected Computational Methods of Solving Simultaneous Equations in Environmental Science for the Treatment of Textile Wastewater

[1]Ojo, S. O., [2]Olayanju, O. K., [3]Bobola, D. G., [4]*Fajobi, A. B., [5]Bolorunduro, K. A., [4]Oke, I. A., and [6]Oni, O.

[1] Computer Science Department, Osun State College of Education, Ilesa, Nigeria
[2] Department of Civil Engineering, Redeemer's University, Ede, Nigeria
[3] PG Student, Department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria
[4] Department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria
[5] Department of Civil Engineering, Federal University, Oye-Ekiti, Nigeria
[6] PG Student Institute of Ecology and Environmental Management, Obafemi Awolowo University, Ile-Ife, Nigeria

*Corresponding Author e-mail: **afajobi@oauife.edu.ng**

Submitted on: 06/09/2022 *Accepted on: 17/03/2023*

Abstract

In this paper, a general model in environmental engineering that relates alum concentration, pH and other chemical treatment factors to the efficiency of chemical treatment techniques was obtained from the literature as a way to evaluate the performance of selected methods of solving simultaneous equations in environmental engineering. Data on textile wastewater treatment (synthetic and typical) were obtained and utilized. Coefficients of model equations were determined using Matrix, Least Square, Gaussian elimination and Microsoft Excel Solver. These final model equations with the determined coefficients were used to compute the performance of the treatment processes and evaluated using standard statistical methods (Total error, mean error, root error, absolute, Model of Selection Criterion (MSC) Model of Selection Criterion (MSC) and mean error). The study revealed that coefficients were between -0.003 for pH to 1.034 for the ratio of coagulant to the concentration of the phosphate. It was revealed that there are significant differences between the coefficients at a 95% confidence level ($F_{15, 45}$. = 27.761; p = 8.39 x 10^{-18}, which is less than 0.05). The tables also revealed that there was no significant difference between the methods at a 95% confidence level ($F_{3, 45}$. = 1.746; p = 0.171, which is greater than 0.05). It was concluded that the order of accuracy of these methods is MES method greater than (>) Matrix > Least square > elimination based on MSC and errors. There is a need to utilize and evaluate other related Excel functions.

Keywords: Preference, Simultaneous, Equations, Substitution, and Elimination.

Introduction

The problem of solving linear and non-linear simultaneous equations is one of the major and vital problems encountered in the fields of environmental science and engineering. Simultaneous equations are well-known scientific and engineering tools for solving practical problems (Bauer and Curran, 2005; Chen, 2013; Xiao *et al.*, 2016). The Laplace, Poisson, and Fourier equations transpire in the solutions of many scientific and engineering problems such as heat flow, pollution control, fluid flow, pipe network analysis, diffusion, sewer design, climate change (Figure 1a), flood control (Figure 1b) and structural problems. The solution of these stated simultaneous equations usually encompasses the solution of a few hundred to a few thousand simultaneous equations (He *et al.*, 2016; Bhattacharjee, 2018; Nagy-Gyorgy *et al.*, 2019). Simultaneous equations can be linear and non-linear, which varies with the field of experts. In addition, elucidations of simultaneous equations in Boolean or Switching algebra is one of the significant topics of investigation on account of the fact that explaining simultaneous Boolean equations is a crucial part of the wide variability of problems [Bhattacharjee, 2018], which include synthesis-simulation, evaluation of digital circuits, identification of initial state, the establishment of finite state sequential network and many others in the area of electrical and electronic engineering. In the specific area of Cryptography (Bhattacharjee, 2018), the need of solving a large-scale system of Boolean equations is vital.

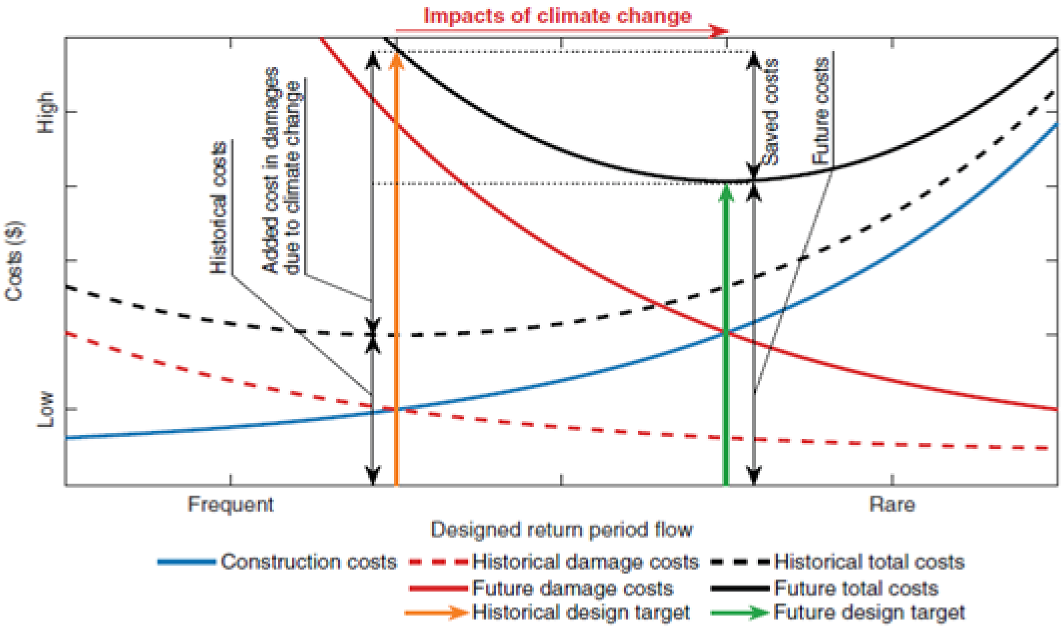

Figure 1a: Conceptualization of climate change impacts on the theoretical design compromise for typical urban infrastructure (Source: Martel *et al.*, 2021)

Figure 1b: Effects of flood on the environment in Lagos, Nigeria

In the areas and fields of modern science and engineering, (Biology, physics, chemistry, computer science, medicine, environmental science, graph theory and pollution control) encountering the problem of solving simultaneous Boolean or other forms of simultaneous equations is a common phenomenon (Loaque and Green, 1991; Petkovic and Herceg, 2001; Petkovic *et al.*, 2007; Bhattacharjee, 2018).

There are a lot of techniques that exist in the traditional literature for solving simultaneous equation problems (In the area of Science and Engineering, the methods of solving simultaneous problems include numerical (Newton, iteration, cross multiplication), graphical, elimination (Newton, Gaussian, Jacobian and Gauss), substitute and least square techniques (Bhattacharjee, 2018). These techniques involve both simple and complex computer programs. The five common techniques studied in the literature. These techniques are Liebman, an explicit method, alternating direction implicit procedure, Iterative alternating direction implicit procedure, matrix inversion procedure and banded matrix inversion procedure. In the application of these techniques, it is established that the technique selected to solve these equations has to be one wherein the capability of the computer, the programmer, and the budget are fitted together.

Many numerical methods are available for the solution of these great numbers of equations, and the authors have had experience in the use of many methods. A solution method of solving the equation may be rated as high by the programmer, but only to discover that the solution time required is not well-suitable for the budget or the computer available. On the other hand, a fast-running solution technique may require several times longer to program and complete, but result in the best solution. Numerous numerical algorithms have been proposed for solving simultaneous equations problems, but there is little or no data on the effectiveness of the matrix inversion technique (Chen, 2013). The main objective of this study is to utilize a matrix, least square, and Microsoft Excel Solver in the determination of the coefficient of the general model parameters (factors) utilising chemical treatment of industrial and synthetic wastewaters in environmental engineering, pollution control and management and to evaluate the performance of these selected factors in predicting phosphate ions removal from the wastewaters.

Materials and Method

A general model equation that relates alum concentration to phosphate removal efficiency of selected wastewater treatment techniques was obtained from the literature (Oke and Okuofu, 2000; Amoko *et al.,* 2016) as follows:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 - \beta_3 X_3 \qquad (1)$$

where: Y is the relative phosphorus removal; β_0 *is the intercept and constant,* β_1*,* β_2 *and* β_3 *are the coefficients of the factors selected;* X_1 *is the ratio of alum dose to phosphate concentration;* X_2 *is the phosphate concentration and* X_3 *is the pH value of the textile wastewater.*

The model equation was transformed and modified into linear regression equations. The experimental data (Tables 1 to 4) was fitted to modified model equations. (Nordin *et al*., 2004; Aber and Sheydaei, 2012; Thirugnanasambandham *et al*., 2014, Asgari *et al.*, 2020; and Bazrafshan *et al.,* 2021). Coefficients of model equations were determined using Matrix, Least Square, Gaussian elimination and Microsoft Excel Solver (Figure 2). Detail procedures for MES are established by Idi *et al.* (2020). These final model equations with the determined coefficients were used to compute the performance of the treatment processes and evaluated using standard statistical methods (Total error, mean error, root error, absolute, Model of Selection Criterion (MSC) Model of Selection Criterion (MSC) and mean error). The Microsoft Excel Solver method was used for the determination of the Coefficients of the model equations based on accuracy, availability at no additional installation and operational costs. The procedure used for the Matrix method can be summarized as follows (Figure 3):

(a) Microsoft Office was installed,
(b) Microsoft Excel was launched
(c) Entered the data
(d) A square Matrix was created;
(e) Understand these Matrix functions (Microsoft Excel provides these matrix functions for calculations, and easy computation purposes)

    (i) MINVERSE is the provider of Invert a matrix
    (ii) MMULT is a function that Multiplies two matrices together
    (iii)MDTERM is a function that can calculate the determinant of a specified matrix array.

(f) Selected the cells for the inverted matrix result for a matrix the same size (square matrix as the original matrix). The use of the function MMINVERSE to invert it.
(g) Specify the array to invert (hit F2, utilise CTRL-SHIFT-ENTER) instead of closing out the function

(h) Multiply matrices using the MMULTI function (selected the cells for the results)
(i) Utilize the MINVERSE function (hit F2, CTRL-SHIFT-ENTER to produce the results)
(j) Copy the results for the variables

The modified model equations are as follows (Nordin *et al.*, 2004; Aber and Sheydaei, 2012; Thirugnanasambandham *et al.*, 2014, Asgari *et al.*, 2020; and Bazrafshan *et al.*, 2021):

$$\sum_{i=1}^{n} Y_i = n\beta_0 + \beta_1 \sum_{i=1}^{n} X_{1i} + \beta_2 \sum_{i=1}^{n} X_{2i} - \beta_3 \sum_{i=1}^{n} X_{3i} \tag{2}$$

$$\sum_{i=1}^{n} X_{1i}Y_i = \beta_0 \sum_{i=1}^{n} X_{1i} + \beta_1 \sum_{i=1}^{n} X_{1i}X_{1i} + \beta_2 \sum_{i=1}^{n} X_{1i}X_{2i} - \beta_3 \sum_{i=1}^{n} X_{1i}X_{3i} \tag{3}$$

$$\sum_{i=1}^{n} X_{2i}Y_i = \beta_0 \sum_{i=1}^{n} X_{2i} + \beta_1 \sum_{i=1}^{n} X_{1i}X_{2i} + \beta_2 \sum_{i=1}^{n} X_{2i}X_{2i} - \beta_3 \sum_{i=1}^{n} X_{2i}X_{3i} \tag{4}$$

Table 1: Chemical Treated 100 % Ortho Phosphate Wastewater

	Y	X1	X2	X3	X4
Equation 2	10.06	12	3.17	5.398	-84.3
Equation 3	2.82	3.17	0.9641	1.49777	-22.279
Equation 4	4.86021	5.398	1.49777	3.025962	-37.4852
Equation 5	70.496	84.3	22.279	37.4852	-593.85

Table 2: Chemical Treated 50:50 Ortho Phosphate wastewaters

	Y	X1	X2	X3	X4
Equation 2	9.18	12	2.69	5.75	-89.2
Equation 3	2.0699	2.69	0.6841	1.1457	-19.902
Equation 4	4.5202	5.75	1.1457	3.3801	-43.454
Equation 5	68.434	89.2	19.902	43.454	-664.24

Table 3: Chemical Treated Textile Wastewater

	Y	X1	X2	X3	X4
Equation 2	7.21	12	18.75	6.01	-83.9
Equation 3	11.5172	18.75	29.6573	9.7325	-131.394
Equation 4	3.9539	6.01	9.7325	3.6707	-42.652
Equation 5	50.708	83.9	131.394	42.652	-587.79

Table 4: Combination of both biological and Chemical Treated Textile wastewater

	Y	X1	X2	X3	X4
Equation 2	11.83	12	32.41	5.65	-90.6
Equation 3	31.9764	32.41	88.2925	14.4974	-244.489
Equation 4	5.5456	5.65	14.4974	3.4775	-42.908
Equation 5	89.286	90.6	244.489	42.908	-685.12

$$\sum_{i=1}^{n} X_{3i}Y_i = \beta_0 \sum_{i=1}^{n} X_{3i} + \beta_1 \sum_{i=1}^{n} X_{3i}X_{1i} + \beta_2 \sum_{i=1}^{n} X_{3i}X_{2i} - \beta_3 \sum_{i=1}^{n} X_{3i}X_{3i} \tag{5}$$

The computations of total error, average error, root total error, absolute error, mean absolute error, Model of Selection Criterion (MSC), and root absolute error are as follows:

$$T_e = \sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2 \tag{6}$$

where; Y_{obsi} is the observed performance; T_e is the total error; n is the total number of the data points calculated, and Y_{cali} is the performance calculated using the method.

$$Av_e = \frac{1}{n} \sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2 \tag{7}$$

where; Av_e is the average error

$$Rt_e = \sqrt{\sum_{i=1}^{n} \left(Y_{obsi} - Y_{cali}\right)^2} \tag{8}$$

where; Rt_e is the root total error

$$Abs_e = \sum_{i=1}^{n} \left(\left|Y_{obsi} - Y_{cali}\right|\right) \tag{9}$$

where; Abs_e is the absolute error

$$Absv_e = \frac{1}{n} \sum_{i=1}^{n} \left(\left|Y_{obsi} - Y_{cali}\right|\right) \tag{10}$$

where; $Absv_e$ is the average absolute error

$$AbsRt_e = \sqrt{\sum_{i=1}^{n} \left(\left|Y_{obsi} - Y_{cali}\right|\right)} \tag{11}$$

where; $AbsRt_e$ is the root absolute error

Model of Selection Criterion can be computed using equation (12) as follows:

$$MSC = \ln\left(\frac{\sum_{i=1}^{N}\left(Y_{obsi} - \overline{Y_{obs}}\right)^2}{\sum_{i=1}^{N}\left(Y_{obsi} - Y_{cali}\right)^2}\right) - \frac{2p}{N} \quad 12)$$

where; $\overline{\mathrm{Y}_{\mathrm{obsi}}}$ *is the average of observed concentration*

$\overline{\mathrm{Y}_{\mathrm{cali}}}$ *is the average of calculated concentration*

p is the number of parameters and N is the number of data points

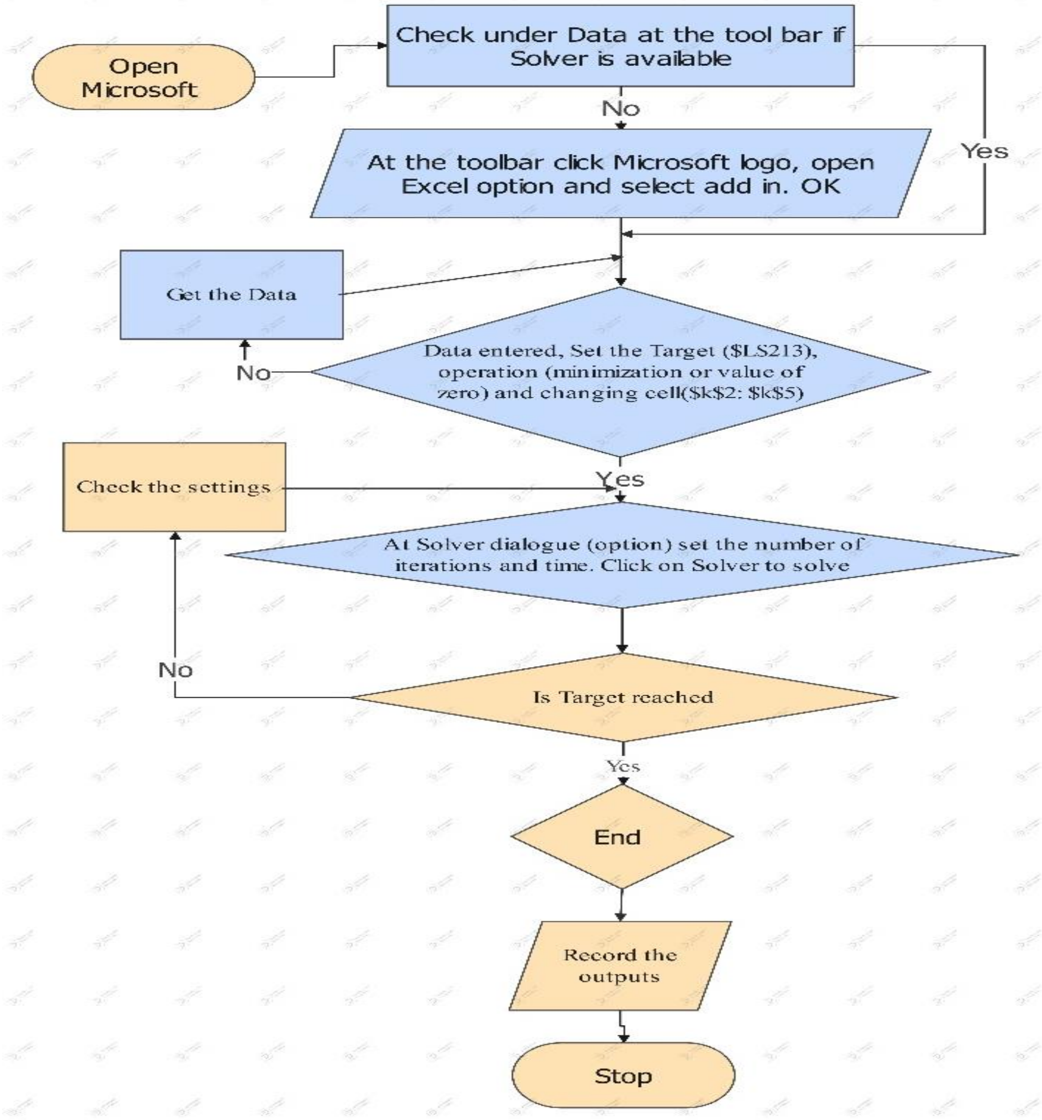

Figure 2: Flowchart for using Microsoft Excel Solver in the computation of the variables

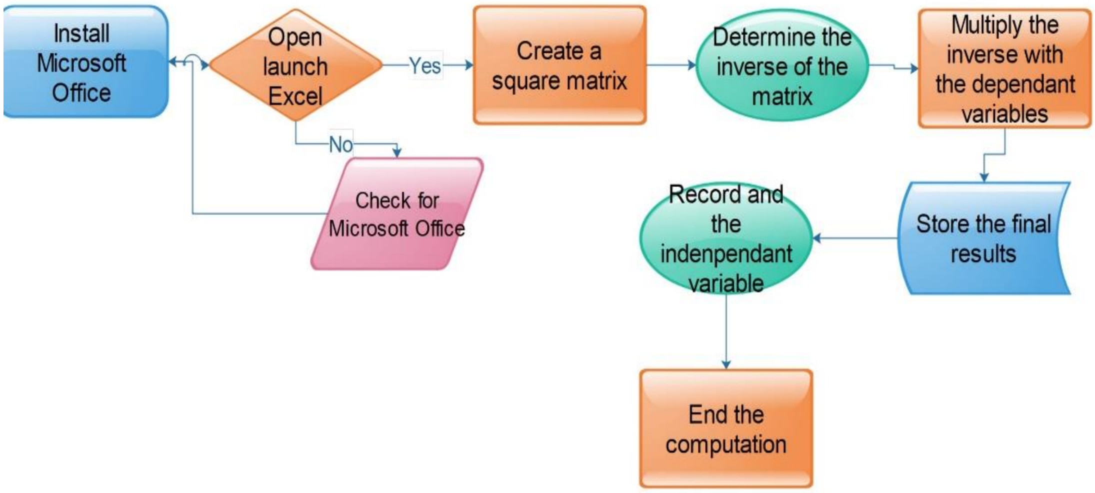

Figure 3: Procedure for using matrix in Microsoft Excel in the computation of the independent variables

Result and Discussion

Table 5 presents the matrix, the inverse of the matrix and the coefficient for chemically treated 100% simulated ortho phosphate wastewater. The Table revealed that the coefficients were between -0.003 for pH to 1.034 for the ratio of coagulant to the concentration of the orthophosphate. The result established that there are two categories of factors for 100% simulated orthophosphate removal from water and textile wastewaters. The two factors are namely positive factors (factors that increase with the performance) and negative factors (a factor that reduces the performance of the removal technique). The table revealed that the higher pH of the textile wastewater reduces the performance of alum (chemical treatment) in removing orthophosphate from textile wastewater. This result indicated that an optimum pH value is required in the treatment of textile wastewater using chemical treatment. The coefficient β_0 was 0.344 (34.4%), which indicated the average performance of the alum treatment without adjustment of the other factors. The coefficient β_2 was 0.439 for the concentration of orthophosphate in the textile wastewater. This result revealed that orthophosphate concentration is a positive factor. The increase in orthophosphate concentrations increases the performance of the chemical treatment of textile wastewaters. This situation can be attributed to higher attraction forces between the coagulant radical (Al^{3+}) and the phosphate radical (PO_5^{3-}). 1.034 for the ratio of coagulant to the concentration of the orthophosphate indicated that the factor is a positive factor of which its influence was similar to orthophosphate concentration.

Table 6 presents the matrix, the inverse of the matrix and the coefficient for chemically treated 50:50 simulated organic: inorganic phosphate wastewater (orthophosphate and polyphosphate). The Table revealed that the coefficients were between 7.913×10^{-5} for pH to 0.827 for the ratio of coagulant to the concentration of the phosphate. The result established that there is only one category of factors for the removal of the phosphate from 50: 50 simulated organic: inorganic phosphate wastewater (Comprehensive data are as presented in Appendices E to G). The result here established that there are two categories of factors for simulated orthophosphate and polyphosphate removal from water and textile wastewaters. The two factors are namely minor positive factors (which is the pH is a factor that increases with a little increase in the performance) and major positive factors (a factor that increases the performance of the removal technique higher than minor factors). The table revealed that the higher pH of the orthophosphate and polyphosphate simulated textile wastewater increases slightly the performance of alum (chemical treatment) in removing orthophosphate and polyphosphate from textile wastewater.

This result indicated that an optimum pH value is required in the treatment of textile wastewater using chemical treatment. The coefficient β_0 was 0.396 (39.6%), which indicated the average performance of the alum treatment without adjustment of the other factors. The presence of the polyphosphate may be attributed to the higher coefficient of β_0 in this case than in the previous case. The coefficient β_2 was 0.384 for the concentration of orthophosphate and polyphosphate in the textile wastewater. This result revealed that orthophosphate and polyphosphate concentration is a positive factor. The increase in orthophosphate and polyphosphate concentrations increases the performance of the chemical treatment of textile wastewaters. This situation can be attributed to higher attraction forces between the coagulant radical (Al^{3+}) and the phosphate radical (PO_5^{3-}). 0.827 for the ratio of coagulant to the concentration of the orthophosphate and polyphosphate indicated that the factor is a positive factor of which its influence was similar to orthophosphate concentration.

Table 7 presents the matrix, the inverse of the matrix and the coefficient for chemically treated textile wastewater. The Table revealed that the coefficients were between 0.033 for pH to 0.399 for the ratio of coagulant to the concentration of the phosphate. The result established that there is only one category of factors for phosphate removal from textile wastewater. The result here established that there are two categories of factors for phosphate removal from typical water and textile wastewaters. The two factors are namely minor positive factors (pH, factors that increase with a little increase in the performance) and major positive factors (a factor that increases the performance of the removal technique higher than minor factors). The table revealed that the higher pH of the typical textile wastewater increases slightly the performance of alum (chemical treatment) in removing phosphate from the textile wastewater. This result indicated that an optimum pH value is required in the treatment of textile wastewater using chemical treatment. The coefficient β_0 was 0.035 (3.50%), which indicated the average performance of the alum treatment without adjustment of the other factors. This lower mean than the previous cases may be attributed to the presence of other pollutants (colour and solids) order than phosphate that inhibited phosphate removal. The coefficient β_2 was 0.344 for the concentration of phosphate in the textile wastewater. This result revealed that phosphate concentration is a positive factor. The increase in phosphate concentrations increases the performance of the chemical treatment of textile wastewaters. This situation can be attributed to higher attraction forces between the coagulant radical (Al^{3+}) and the phosphate radical (PO_5^{3-}). 0.399 for the ratio of coagulant to the concentration of the phosphate indicated that the factor is a positive factor of which its influence was similar to orthophosphate concentration.

Table 5: Matrix, inverse and coefficient for Chemical Treated 100 % Orthophosphate Wastewater

	Y	X1	X2	X3	X4	Inverse				Variable and Value	
Equation1	10.06	12	3.17	5.398	-84.3	41.39	1.17	-4.98	-5.60	β_0	0.344
Equation 2	2.82	3.17	0.9641	1.49777	-22.279	1.17	8.68	-1.34	-0.41	β_1	1.034
Equation 3	4.86021	5.398	1.49777	3.025962	-37.4852	-4.98	-1.34	2.28	0.61	β_2	0.439
Equation 4	70.496	84.3	22.279	37.4852	-593.85	5.60	0.41	-0.61	-0.77	β_3	-0.003

Table 6: Matrix, inverse and coefficient for Chemical Treated 50:50 Orthophosphate wastewaters

	Y	X1	X2	X3	X4	Inverse				Variable and Value	
Equation1	9.18	12	2.69	5.75	-89.2	152.41	26.80	32.01	-23.36	β_0	0.396
Equation 2	2.0699	2.69	0.6841	1.1457	-19.902	26.80	28.32	12.49	-5.26	β_1	0.827
Equation 3	4.5202	5.75	1.1457	3.3801	-43.454	32.01	12.49	10.58	-5.37	β_2	0.384
Equation 4	68.434	89.2	19.902	43.454	-664.24	23.36	5.26	5.37	-3.65	β_3	7.91×10^{-5}

Table 7: Matrix, inverse and coefficient for Chemical Treated Textile Wastewater

	Y	X1	X2	X3	X4	Inverse				Variable and Value	
Equation1	7.21	12	18.75	6.01	-83.9	87.11	8.79	14.84	11.55	β_0	0.035
Equation 2	11.5172	18.75	29.6573	9.7325	131.394	-8.79	5.49	-3.08	0.25	β_1	0.399
Equation 3	3.9539	6.01	9.7325	3.6707	-42.652	14.84	-3.08	4.81	-1.78	β_2	0.344
Equation 4	50.708	83.9	131.394	42.652	-587.79	11.55	-0.25	1.78	-1.72	β_3	0.033

Table 8 presents the matrix, the inverse of the matrix and the coefficient for the combination of biologically and chemically treated textile wastewater. The Table revealed that the coefficients were between 0.025 for pH to 0.952 for the constant of the model equation of the phosphate removal. The result established that there is only one category of factors for phosphate removal from typical textile wastewater. The presence of pretreatment using biological treatment improved the performance of the treatment technique, which indicated that in practice pretreatment of textile wastewater is required for an effective treatment process.

Table 8: Matrix, inverse and coefficient for Combination of biological and Chemical Treated Textile wastewater

	Y	X1	X2	X3	X4	Inverse				Variable and Value	
Equation 1	11.83	12	32.41	5.65	-90.6	239.61	-62.93	-57.55	-5.62	β_0	0.952
Equation 2	31.9764	32.41	88.2925	14.4974	-244.489	-62.93	21.39	20.13	-0.57	β_1	0.074
Equation 3	5.5456	5.65	14.4974	3.4775	-42.908	-57.55	20.13	20.26	-0.84	β_2	0.046
Equation 4	89.286	90.6	244.489	42.908	-685.12	5.62	0.57	0.84	-1.00	β_3	0.025

Table 9 presents the coefficients of the factors for all the methods used while Table 10 provides information on the result of the ANOVA conducted on the coefficients. These tables revealed that there are significant differences between the coefficients at a 95 % confidence level ($F_{15, 45}$. = 27.761; p = 8.39 x 10^{-18}, which is less than 0.05). The tables also revealed that there was no significant difference between the methods at a 95 % confidence level ($F_{3, 45}$. = 1.746; p = 0.171, which is greater than 0.05).

Table 9: Coefficient of the factors for the process using the four methods

		MES	Elimination	Least Square	Matrix
Treated (Industrial Wastewater Treated using Biological and Chemical Treatment)	β_0	0.952	0.622	1.134	0.952
	β_1	0.074	0.074	0.023	0.074
	β_2	0.046	0.046	0.016	0.046
	β_3	0.025	0.025	0.027	0.025
Ortho Phosphate: Synthetic Wastewater, 100 % ortho-phosphate) Treated using Chemical treatment only)	β_0	0.344	0.782	0.319	0.344
	β_1	1.034	1.034	1.123	1.034
	β_2	0.439	0.439	0.492	0.439
	β_3	-0.003	-0.003	-0.001	-0.003
Industrial Wastewater Treated using Chemical treatment only	β_0	0.035	-0.077	0.227	0.035
	β_1	0.399	0.399	0.389	0.399
	β_2	0.344	0.344	0.370	0.344
	β_3	0.033	0.033	0.060	0.033
Synthetic Wastewater (50 % ortho-phosphate and 50% poly phosphate) Treated using Chemical treatment only	β_0	-0.161	0.769	0.547	0.396
	β_1	0.701	0.827	0.863	0.827
	β_2	0.256	0.384	0.417	0.384
	β_3	-0.087	0.000	0.024	7.91 x 10^{-5}

Table 10: The result of the ANOVA performed on coefficient of the factors

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value	F critical
Within Coefficients	7.086711	15	0.47244743	27.76082	8.39 x 10^{-18}	1.894875
Between the Methods	0.089161	3	0.02972019	1.746347	0.171074	2.811544
Error	0.765832	45	0.017018495			
Total	7.941704	63				

These differences between the coefficients for the factors and different wastewaters can be attributed to the composition of the wastewater, reactions between the coagulant, other elements present and phosphate concentration. The higher removal coefficients of orthophosphate might be explained by the higher reactivity of orthophosphate when the phosphate precipitation occurs. In addition, the size of phosphorus species and the electrolyte concentration mainly influence on the coefficients of phosphorus using coagulation. It is known that orthophosphate has a smaller ionic radius and higher negative charge per phosphorus atom than polyphosphate. This ionic radius plays a significant role in phosphorus removal from aqueous solutions (wastewater included). In the case of polyphosphate, it contains phosphorus atoms, which are in the fully oxidized state. This reason makes polyphosphate reasonably stable in chemical reactions (Park *et al.*, 2016). The indication of these results is that wastewater containing orthophosphate can be treated more easily using a coagulation process than wastewater containing polyphosphate. The environmental implications are that the eutrophication in lakes, ponds and lagoons can be reduced through orthophosphate ions removal by coagulation. Furthermore, larger amounts of coagulants and electrolytes in the aqueous solution promote destabilization of the system, and reactive orthophosphate, which might be easily removed during the coagulation process when the coagulant concentration was high (Appendices A to D provide a piece of detailed information on the performance coagulation as a process in removing phosphate from varieties of aqueous solutions). In environmental pollution control and management, it can be highlighted that textile wastewater, institution wastewater and domestic wastewater which are known to contain both orthophosphate and polyphosphate can be treated utilizing both biological and chemical treatment processes for an effective removal of phosphates.

Accuracy Attained by the Methods

Figures 4 to 7 and Tables 11 to 14 present the performance of these methods in predicting the observed performance. Figures 4 to 7 show that the predictions from MES (Solver method), Least square and Matrix methods are very close to the observed performance in the treatment options. These figures show that in all the cases predictions of the performance based on the values of coefficients obtained using elimination were higher than the predictions based on the coefficients from other methods. It was also observed that the predictions of the performances using coefficients from other methods were closer to the observed experimental performances, which indicated that care must be taken in the utilization of the elimination method. These results and observations support the general observations and statements on the accuracy of numerical methods in solving engineering problems. In addition, it can be observed from the figures that the accuracy of the elimination method was lower than the other three methods in all the cases. This lower accuracy by elimination method can be attributed to truncation error and other related errors. Tables 11 to 14 present the values for total error, mean (average) error, root error, MSC, total absolute error, mean absolute error, and root absolute error. In all the treatment and prediction options elimination method produced the highest error (4.681) and the lowest MSC (-3.862). The tables also show that the Solver method had the lowest total error of 0.006, mean error of 0.001 and the highest MSC of 3.455. Next to the MES method are the Matrix and Least square methods in the same order. These results indicated the order of the accuracy of these methods is MES method greater than (>) Matrix > Least square > elimination. This order of accuracy (based on lower errors and higher MSC) revealed that the utilization of more functions of Excel (INTERCEPT, MOD.MULT, MODE>SNGL, MUNIT, PRODUCT, TRANSPOSE, SolverAdd, SolverFinish, SolverGet and SolverSolve) in solving simulation equation should be employed as these functions are available at no additional cost.

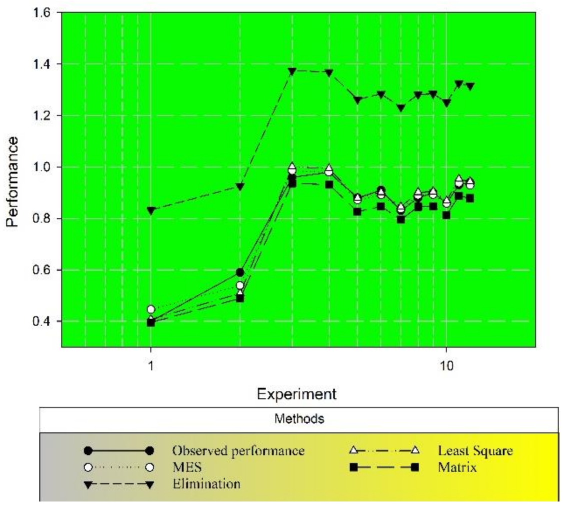

Figure 4: Relationship between the outputs of the methods and observed performance for 100 % orthophosphate wastewaters.

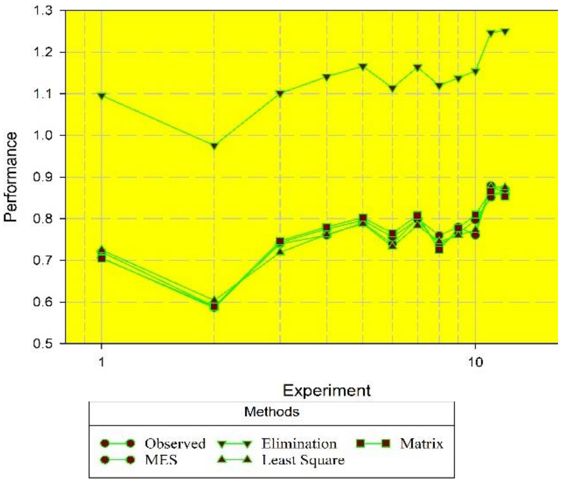

Figure 5: Relationship between the outputs of the methods and observed performance for 50 orthophosphates: 50 polyphosphate wastewaters.

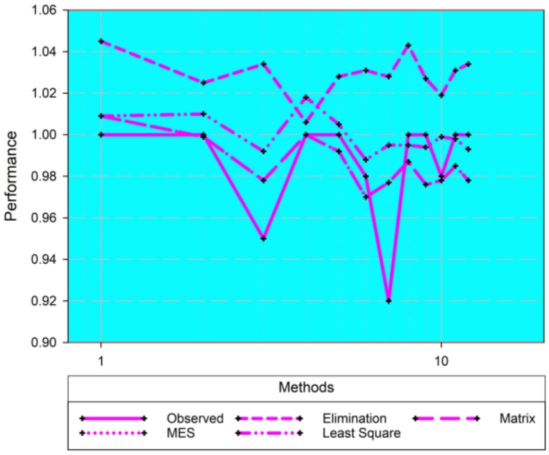

Figure 6: Relationship between the outputs of the methods and observed performance for both biological and chemical treatment of textile wastewaters

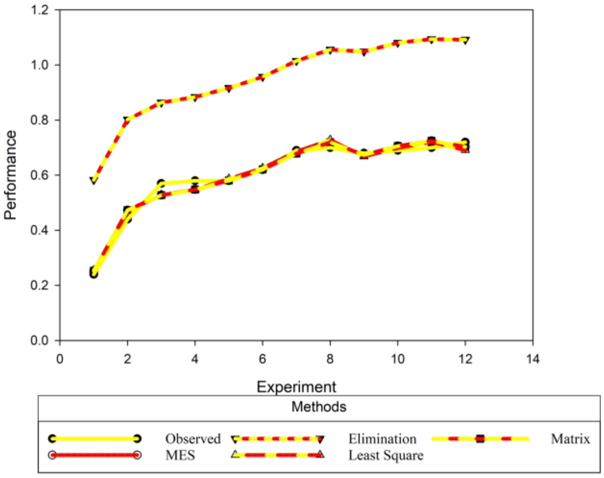

Figure 7: Relationship between the outputs of the methods and observed performance for chemical treatment of textile wastewaters

Table 11: Statistical of the methods (Ortho Phosphates)

Evaluation	Solver	Elimination	Least Sq.	Matrix
Total Error	0.006	1.832	0.010	0.008
Mean Error	0.001	0.153	0.001	0.001
Root Error	0.078	1.354	0.099	0.099
MSC	3.455	-2.244	2.978	3.395
Absolute Error				
Total Error	0.192	4.681	0.228	0.194
Mean Error	0.016	0.390	0.019	0.016
Root Error	0.438	2.164	0.478	0.441

Table 12: Statistical of the methods (A 50:50)

Evaluation	Solver	Elimination	Least Sq.	Matrix
Total Error	0.004	1.682	0.002	0.005
Mean Error	0.000	0.140	0.000	0.000
Root Error	0.062	1.297	0.045	0.071
MSC	2.235	-3.862	2.877	2.189
Absolute Error				
Total Error	0.174	4.490	0.134	0.176
Mean Error	0.014	0.374	0.011	0.015
Root Error	0.417	2.119	0.366	0.420

Table 13: Evaluation of the methods: Untreated Textile (Chemical Treatment only)

Evaluation	Solver	Elimination	Least Sq.	Matrix
Total Error	0.006	1.471	0.006	0.009
Mean Error	0.000	0.123	0.001	0.001
Root Error	0.076	1.213	0.079	0.094
MSC	3.118	-2.426	3.043	3.072
Absolute Error				
Total Error	0.217	4.185	0.234	0.219
Mean Error	0.018	0.349	0.019	0.018
Root Error	0.466	2.046	0.484	0.472

Table 14: Evaluation of the methods: Treated Textile (Biological and Chemical)

Evaluation	Solver	Elimination	Least Sq.	Matrix
Total Error	0.006	0.031	0.008	0.009
Mean Error	0.000	0.003	0.001	0.001
Root Error	0.076	0.176	0.092	0.094
MSC	-0.269	-1.947	-0.646	-0.315
Absolute Error				
Total Error	0.189	0.520	0.207	0.191
Mean Error	0.016	0.043	0.017	0.016
Root Error	0.435	0.721	0.455	0.441

Conclusions

In this study, a general model in environmental engineering that relates alum concentration, pH and other chemical treatment factors to the efficiency of chemical treatment techniques was obtained from the literature as a way to evaluate the performance of selected methods of solving simultaneous equations in environmental engineering. Data on textile wastewater treatment (synthetic and typical) were obtained and utilized. The study revealed that the higher pH of the textile wastewater reduces the performance of alum (chemical treatment) in removing orthophosphate from textile wastewater. This result indicated that an optimum pH value is required in the treatment of textile wastewater using chemical treatment.

For the evaluation of the accuracy of methods, it was in the order of MES method greater than (>) Matrix > Least square > elimination based on MSC and errorThe matrix method was accurate and faster than other methods (MMULT and MINVERSE are the two functions required. The two functions are available on Microsoft Excel). MES and Least squares are the next method, but required higher skills in computing. Although, elimination can be used, but the accuracy was found to be lower than any other method. It was recommended that there is a need to evaluate elimination methods further with Excel functions.

References

Aber, S and Sheydaei, M. (2012). Removal of COD from Industrial Effluent Containing Indigo Dye Using Adsorption Method by Activated Carbon Cloth: Optimization, Kinetic, and Isotherm Studies. Clean – Soil, Air, Water 40 (1), 87–94

Amoko, J.S; Fehintola E. O; Obijole, O. A and Oke, I. A. (2016). Mechanisms and modelling phosphate removal from textile wastewaters by chemical coagulation. Direct Research Journal of Chemistry and Material Science. 4(1), 1 – 13

Asgari, G., Shabanloo, A, Salari, M and Eslami, F (2020). Sonophotocatalytic treatment of AB113 dye and real textile wastewater using ZnO/persulfate: Modeling by response surface methodology and artificial neural network. Environmental Research 184, 109367

Bauer, D. J., & Curran, P. J. (2005). *Probing Interactions in Fixed and Multilevel Regression: Inferential and Graphical Techniques. Multivariate Behavioral Research, 40(3), 373–400.*

Bazrafshan, E, Zarei, A. A., Mohammadi, L, Zafar, M. N, Foroughi, M., Aman, S., Sabri, F., Mahvi, A. H., Barahuie, F, Zafar, M (2021). Efficient tetracycline removal from aqueous solutions using ionic liquid modified magnetic activated carbon (IL@mAC). Journal of Environmental Chemical Engineering 9 (2021) 106570

Bhattacharjee, P. R. (2018). A Novel Method for Solving Simultaneous Equations in Boolean/Switching Algebra. IETE Journal of Education, 59(1), 18–25. doi:10.1080/09747338.2018.1465364

Chen, K. (2013). Implicit dynamic system for online simultaneous linear equations solving. Electronics Letters, 49(2), 101–102.

He, Z.-H., Agudelo, O. M., Wang, Q.-W., and De Moor, B. (2016). Two-sided coupled generalized Sylvester matrix equations solving using a simultaneous decomposition for fifteen matrices. Linear Algebra and Its Applications, 496, 549–593.

Idi, M. D. Aladesanmi, T. A. Lukman, S.. Asani, M. A, Usman, Y. M and Oke, I.A. (2020). A Numerical Formula And Cost Benefit Of Groundwater Recharge In Maiduguri, Nigeria . (FUTA Journal of Technology, Akure). FUTAJEET, 14(2), 126 -138

Loague, K., & Green, R. E. (1991). *Statistical and graphical methods for evaluating solute transport models: Overview and application. Journal of Contaminant Hydrology, 7(1-2), 51–73.* doi:10.1016/0169-7722(91)90038-3

Martel, J.L; Brissette, F. P.; Lucas-Picher, P.; Troin, M.; and Arsenault, R. (2021). Climate Change and Rainfall Intensity–Duration–Frequency Curves: Overview of Science and Guidelines for Adaptation. ASCE J. Hydrol. Eng.,26(10): 03121001

Nagy-György, P., Hős, C. (2019) A Graphical Technique for Solving the Couette-Poiseuille Problem for Generalized Newtonian Fluids", Periodica Polytechnica Chemical Engineering, 63(1), pp. 200–209, 2019. https://doi.org/10.3311/PPch.11817

Nordin M. Y, Venkatesh VC, Sharif S, Elting S, Abdullah A. (2004). Application of response surface methodology in describing the performance of coated carbide tools when turning AISI 104 steel. J Mater Process Technol 145: 46–58.

Oke, I. A and Okuofu, C.A (2000). Mathematical modeling of Phosphorous Removal from Textile Wastewater With Aluminum Sulphate (Alum; $Al_2(So_4)_3$– YH_2O).Nigerian Journal of Engineering, 8 (2), 59- 67

Park, T, Ampunan, V.; Lee, S. and Chung, E. (2016). Chemical behavior of different species of phosphorus in coagulation. Chemosphere 144, 2264 - 2269

Petković, M. S., and Herceg, Đ. (2001). Point estimation of simultaneous methods for solving polynomial equations: a survey. Journal of Computational and Applied Mathematics, 136(1-2), 283–307.

Petković, M. S., Petković, L. D., and Rančić, L. Z. (2007). Point estimation of simultaneous methods for solving polynomial equations: A survey (II). Journal of Computational and Applied Mathematics, 205(1), 32–52.

Thirugnanasambandham, K., Sivakumar, V., and Prakash, J. M. (2014). Response surface modelling and optimization of treatment of meat industry wastewater using electrochemical treatment method. Journal of the Taiwan Institute of Chemical Engineers 39, 432–456

Xiao, L., Liao, B., Jin, J., Lu, R., Yang, X., and Ding, L. (2016). A finite-time convergent dynamic system for solving online simultaneous linear equations. International Journal of Computer Mathematics, 94(9), 1778–1786.

Appendices

Appendix A: Performance of Chemical Treatment on Synthetic Wastewaters and Values of The Selected Factors

	Ortho Phosphate				A 50:50 Phosphate			
Serial Number	Observed Efficacy	Beta	Pi	pH	Observed Efficacy	Beta	Pi	pH
1	0.40	0.07	0.009	7.4	0.72	0.39	0.01	6.8
2	0.59	0.16	0.009	7.4	0.59	0.12	0.28	6.8
3	0.96	0.44	0.37	7.2	0.74	0.27	0.28	7.4
4	0.98	0.46	0.31	7.2	0.76	0.3	0.32	7.4
5	0.88	0.28	0.49	6.8	0.79	0.33	0.32	7.4
6	0.91	0.3	0.49	6.8	0.74	0.18	0.51	7.6
7	0.83	0.21	0.58	6.2	0.8	0.24	0.51	7.6
8	0.88	0.26	0.58	6.8	0.76	0.14	0.61	7.4
9	0.9	0.25	0.61	6.8	0.78	0.13	0.68	7.6
10	0.86	0.2	0.65	6.8	0.76	0.15	0.68	7.8
11	0.93	0.27	0.66	7.4	0.88	0.22	0.77	7.6
12	0.94	0.27	0.64	7.5	0.86	0.22	0.78	7.8

Appendix B: Performance of the treatment processes on both textile wastewaters and values of the selected factors

	Chemical Only (Untreated Textile)				Biological and Chemical (Treated Textile)			
Serial Number	Observed Efficacy	Beta	Pi	pH	Observed Efficacy	Beta	Pi	pH
1	0.24	1.03	0.08	6.7	1.00	3.24	0.03	7.4
2	0.44	1.58	0.08	6.7	1.00	2.91	0.23	7.2
3	0.57	1.52	0.32	6.8	0.95	2.78	0.32	7.8
4	0.58	1.57	0.32	6.8	1.00	2.65	0.45	6.8
5	0.58	1.49	0.51	6.8	1.00	2.68	0.56	7.4
6	0.62	1.59	0.51	6.8	0.98	2.45	0.72	7.9
7	0.69	1.63	0.63	6.8	0.92	2.48	0.72	7.7
8	0.7	1.69	0.68	6.8	1.00	2.9	0.32	7.8
9	0.68	1.59	0.72	7.4	1.00	2.44	0.76	7.7
10	0.69	1.67	0.72	7.4	0.98	2.36	0.82	7.5
11	0.7	1.71	0.72	7.3	1.00	2.95	0.08	7.6
12	0.72	1.68	0.72	7.6	1.00	2.57	0.64	7.8

Appendices E to H: Computation of the coefficients using the Matrix Method

Appendix E

X1	X2	X3	X4	Y
12	3.17	5.398	-84.3	10.06
3.17	0.9641	1.49777	-22.279	2.82
5.398	1.49777	3.025962	-37.4852	4.86021
84.3	22.279	37.4852	-593.85	70.496
Inverse				
41.39	1.17	-4.98	-5.60	
1.17	8.68	-1.34	-0.41	
-4.98	-1.34	2.28	0.61	
5.60	0.41	-0.61	-0.77	
Value				
K1	0.344			
K2	1.034			
K3	0.439			
K4	-0.003			

Appendix F

X1	X2	X3	X4	Y
12	2.69	5.75	-89.2	9.18
2.69	0.6841	1.1457	-19.902	2.0699
5.75	1.1457	3.3801	-43.454	4.5202
89.2	19.902	43.454	-664.24	68.434
Inverse				
152.41	26.80	32.01	-23.36	
26.80	28.32	12.49	-5.26	
32.01	12.49	10.58	-5.37	
23.36	5.26	5.37	-3.65	
Value				
K1	0.396			
K2	0.827			
K3	0.384			
K4	7.91 X 10^{-05}			

Appendix G

X1	X2	X3	X4	Y
12	18.75	6.01	-83.9	7.21
18.75	29.6573	9.7325	-131.394	11.5172
6.01	9.7325	3.6707	-42.652	3.9539
83.9	131.394	42.652	-587.79	50.708
Inverse				
87.11	-8.79	14.84	-11.55	
-8.79	5.49	-3.08	0.25	
14.84	-3.08	4.81	-1.78	
11.55	-0.25	1.78	-1.72	
Value				
K1	0.035			
K2	0.399			
K3	0.344			
K4	0.033			

Appendix H

X1	X2	X3	X4	Y
12	32.41	5.65	-90.6	11.83
32.41	88.2925	14.4974	-244.489	31.9764
5.65	14.4974	3.4775	-42.908	5.5456
90.6	244.489	42.908	-685.12	89.286
Inverse				
239.61	-62.93	-57.55	-5.62	
-62.93	21.39	20.13	-0.57	
-57.55	20.13	20.26	-0.84	
5.62	0.57	0.84	-1.00	
Value				
K1	0.952			
K2	0.074			
K3	0.046			
K4	0.025			